USC Viterbi School of Engineering

Computation and Data-Driven Discovery group

Circumventing the Solution of Inverse Problems in Mechanics through Deep Learning

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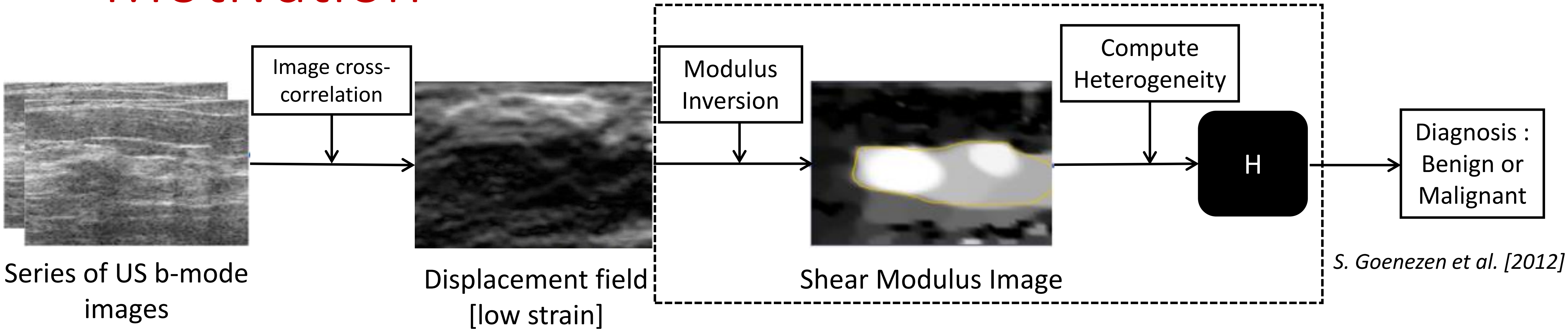
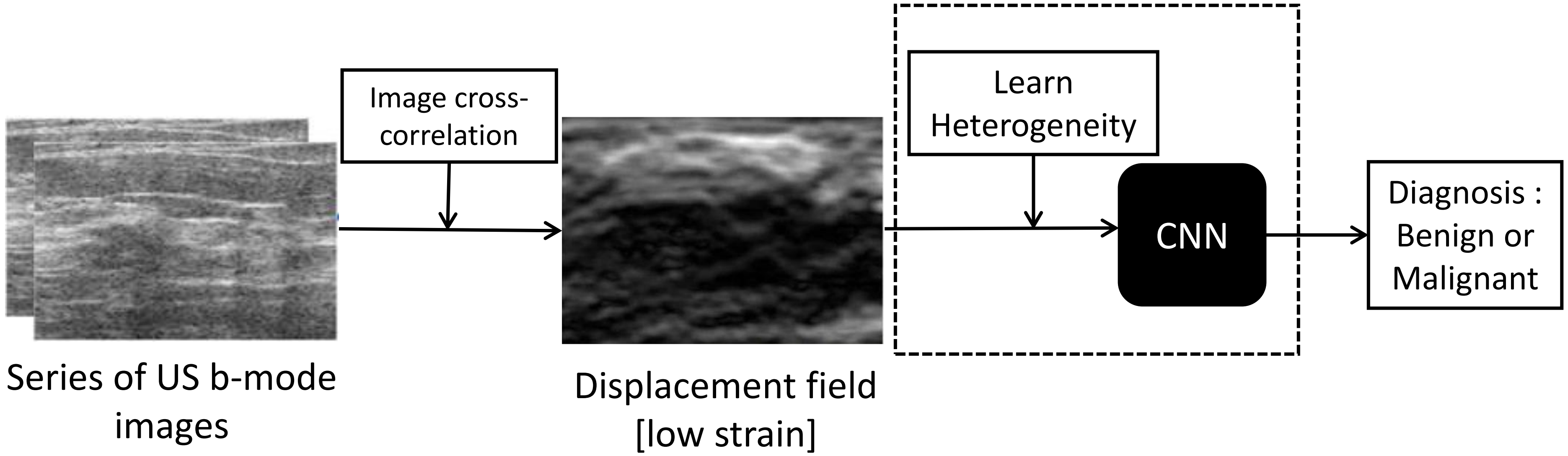
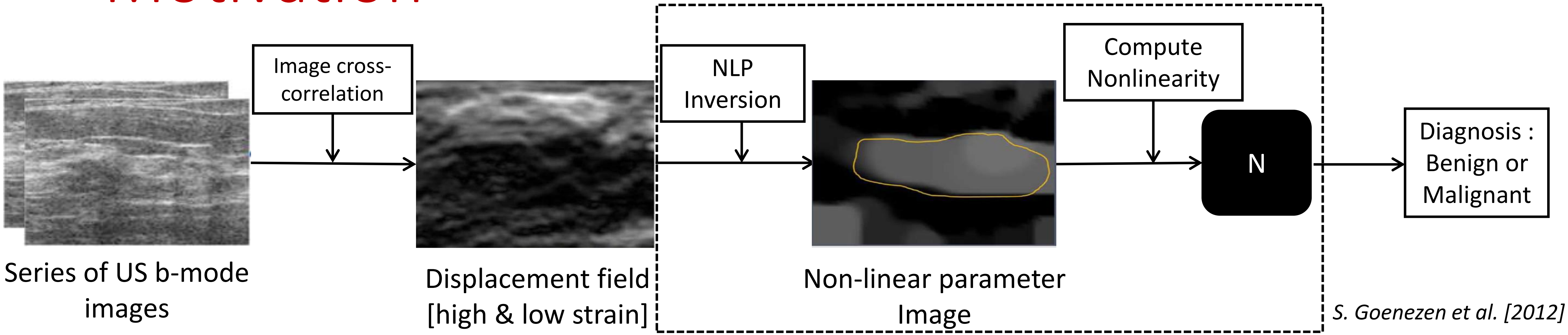
Motivation



- Requires the solution of an inverse problem with many optimization parameters(~10^4) – Time consuming and expensive.

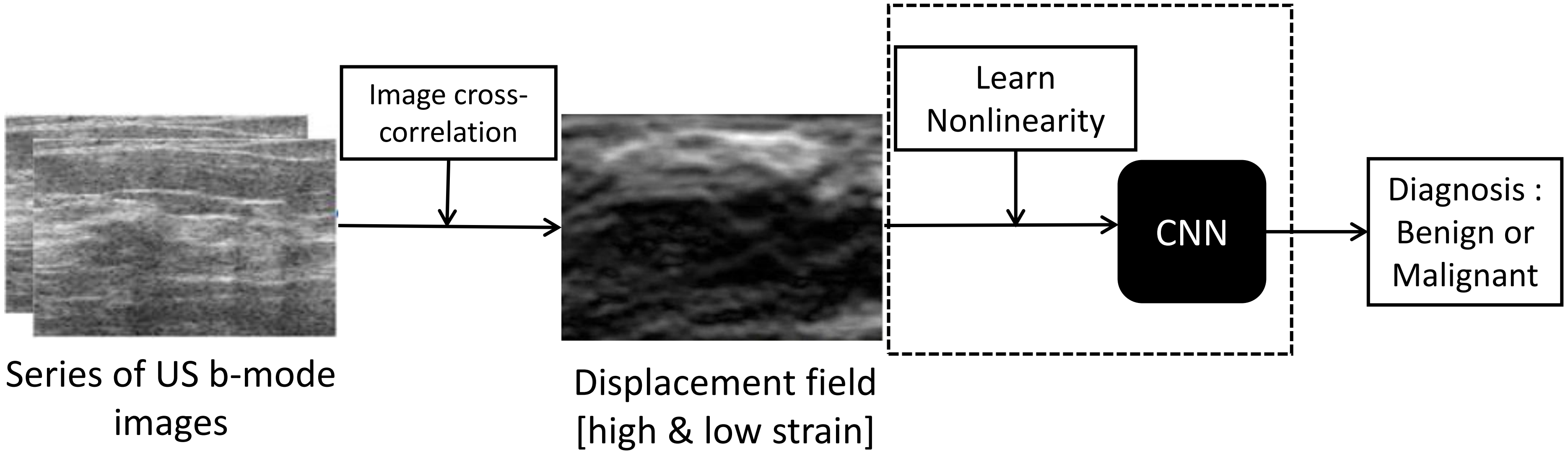


Motivation



S. Goenezen et al. [2012]

- Although NLP has rich diagnostic value, solution of an inverse problem is even more difficult due to high strain involved.



Outline

- Motivation
- Introduction and problem setup
- Classification based on Heterogeneity in Shear Modulus
- Classification based on Nonlinear elastic behavior
- Conclusion

What is Learning?

[Supervised learning]

Input (X)

Output (Y)

[Data]

Learning Algorithm

$Y = f(X;\theta_1, \theta_2,....,\theta_n)$

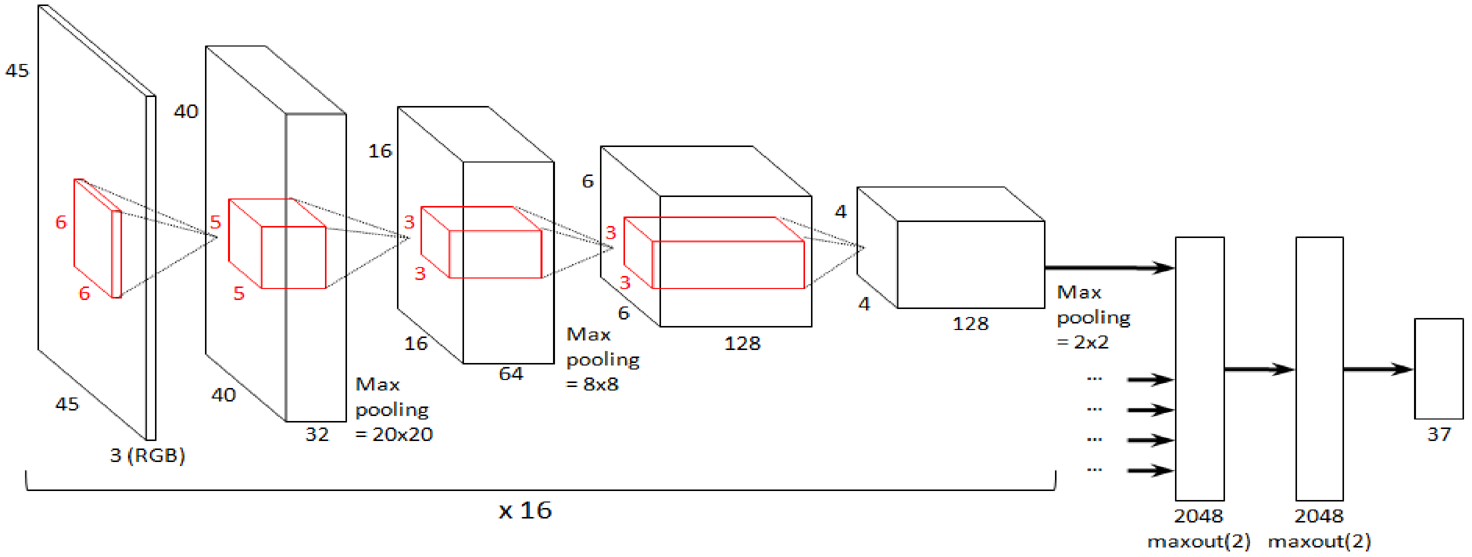
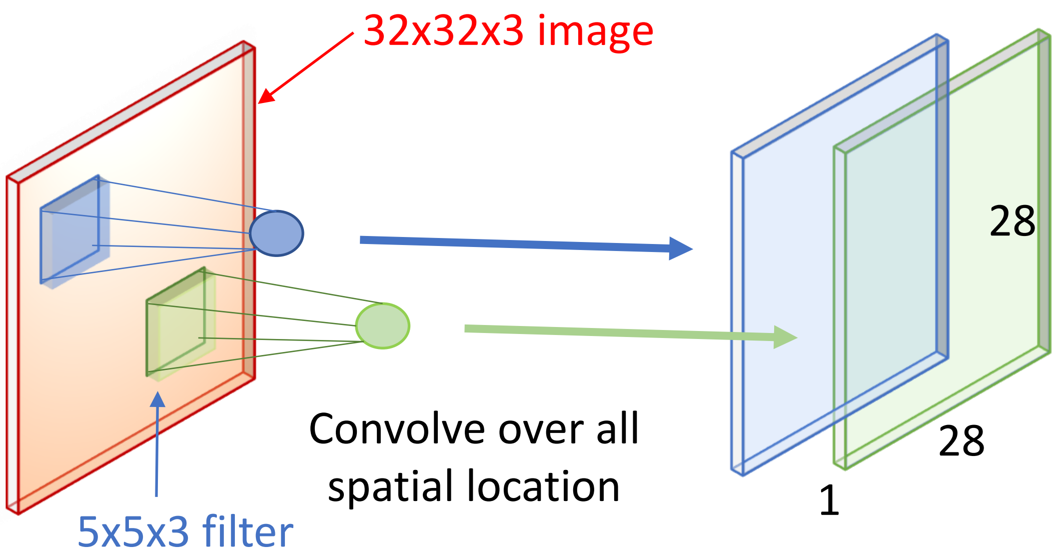


Image → Stage 1 → Stage 2 - - - → Stage n → Fully connected Layer → Class Label

Conv. → Activation → Pooling

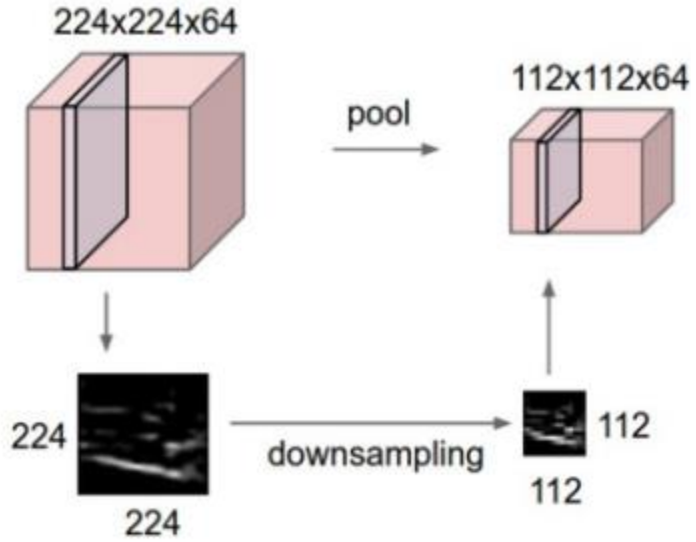
Convolution Layer



Activation Layer

- Goal of activation layer is to introduce some non-linearity to the activation map.
- Commonly used activation functions : ReLU, Leaky ReLU and Sigmoid.

Pooling Layer



Goal

- Reduce the dimensionality – computational gain and less chance of overfit.
- Introduce some translational invariance.

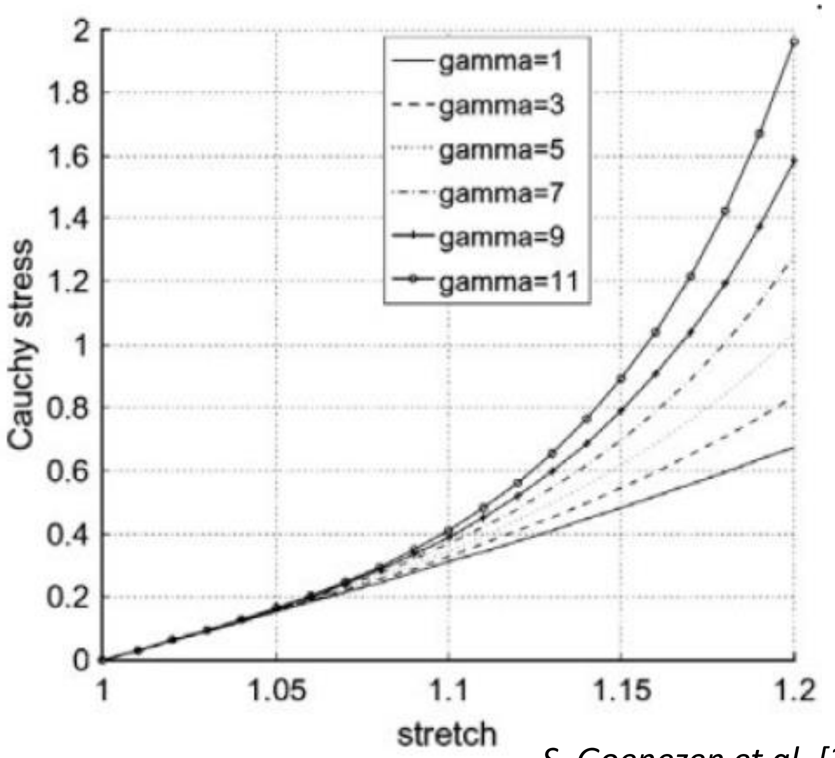
Types of pooling

- Max pooling
- Average pooling

Problem Setup

We model the tissue as an incompressible isotropic hyper-elastic solid with strain energy density function given by:

$$W = \frac{\mu}{2\gamma}\left[e^{\gamma\left(J^{\frac{-2}{3}}I_1-3\right)} - 1\right]$$



S. Goenezen et al. [2011]

- μ represents the slope of the stress-strain curve at zero strain.
- γ represents the nonlinear elastic response of the material

Heterogeneity study

L. Tiangxiao [2015]

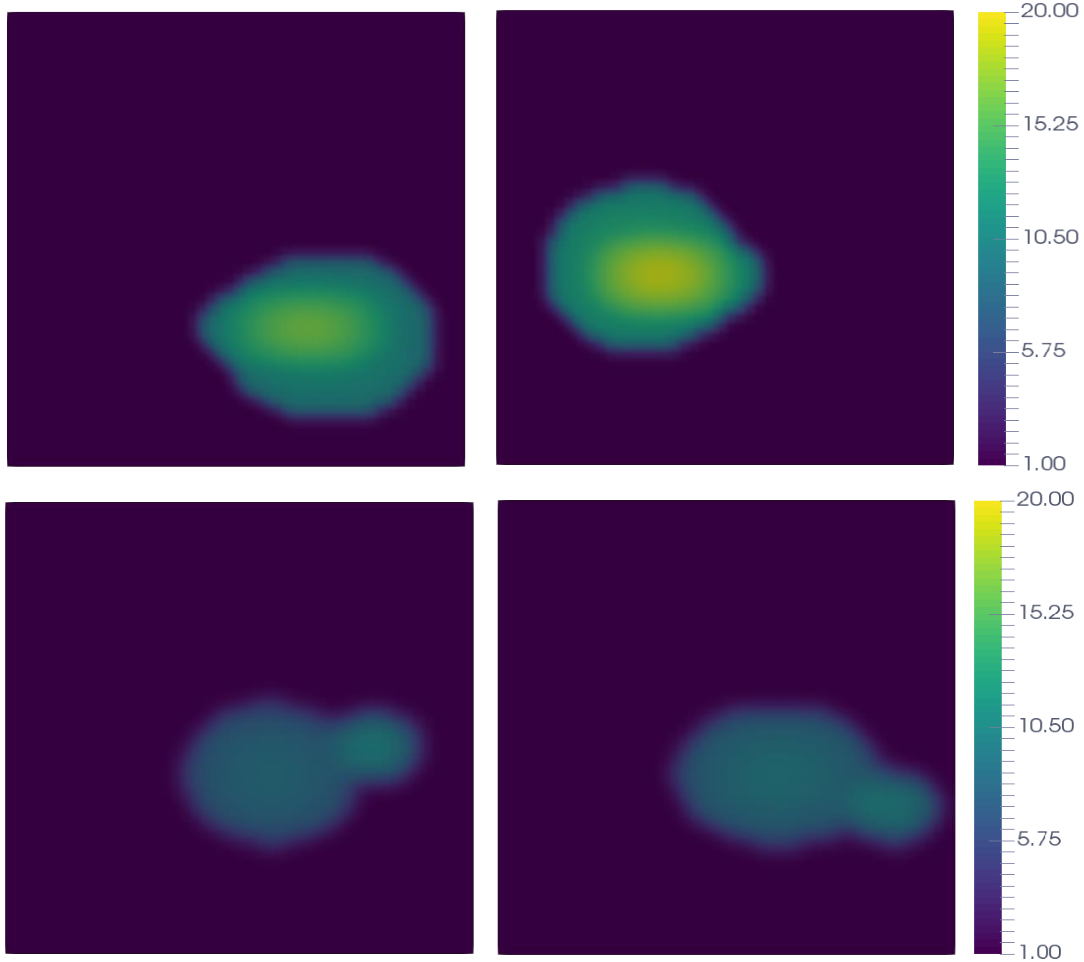
Malignant tumors are characterized by more heterogenous distribution of shear modulus than their benign counterparts.

Problem setup

Heterogeneity study

Benign

Malignant



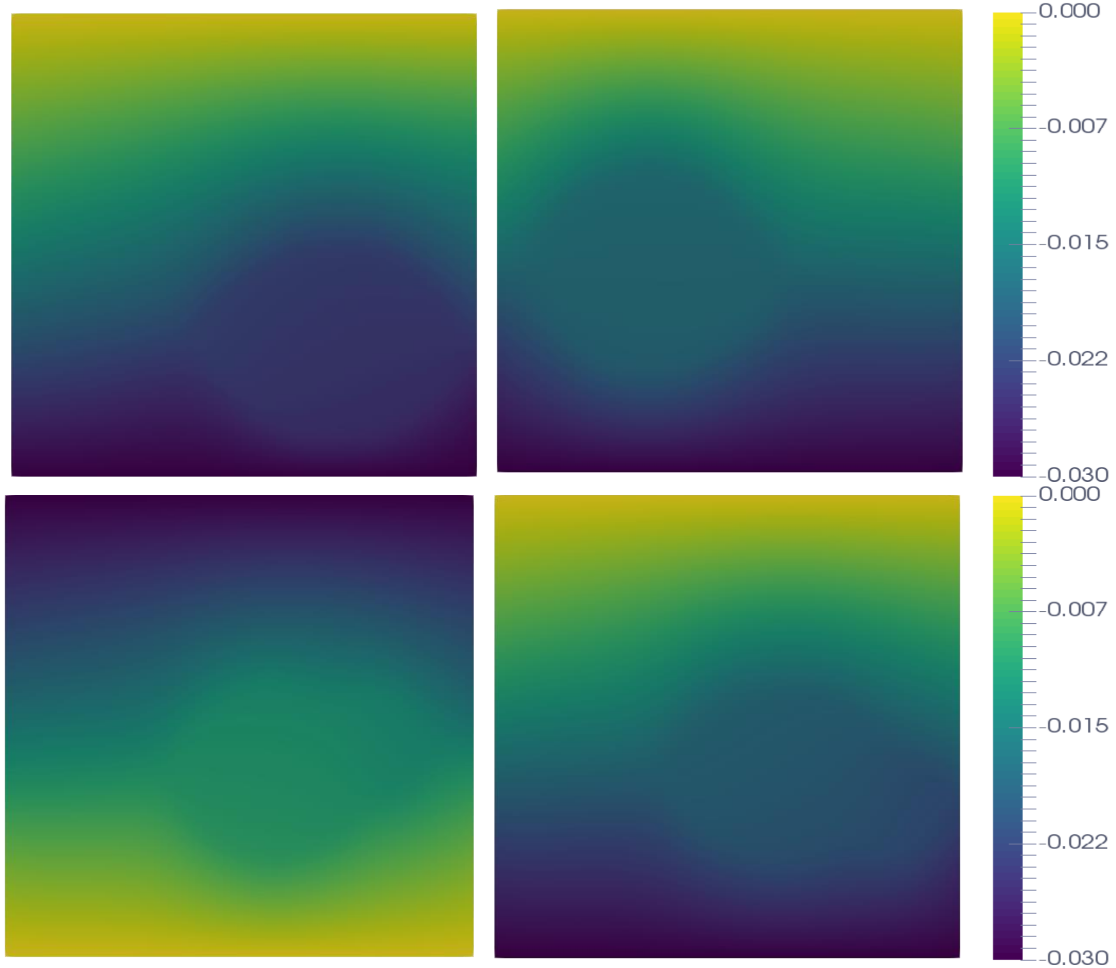
- For both cases, the SM is represented as a superposition of Gaussian distribution. For malignant tumors, the centers of the Gaussians are separated by certain distance to ensure two foci.
- 10,000 distinct shear modulus distributions were generated (5000 each) by changing the shape of the tumor, location of the tumor and the value of the shear modulus.

Problem setup

Generating *in-silico* displacement field [Heterogeneity study]

Benign

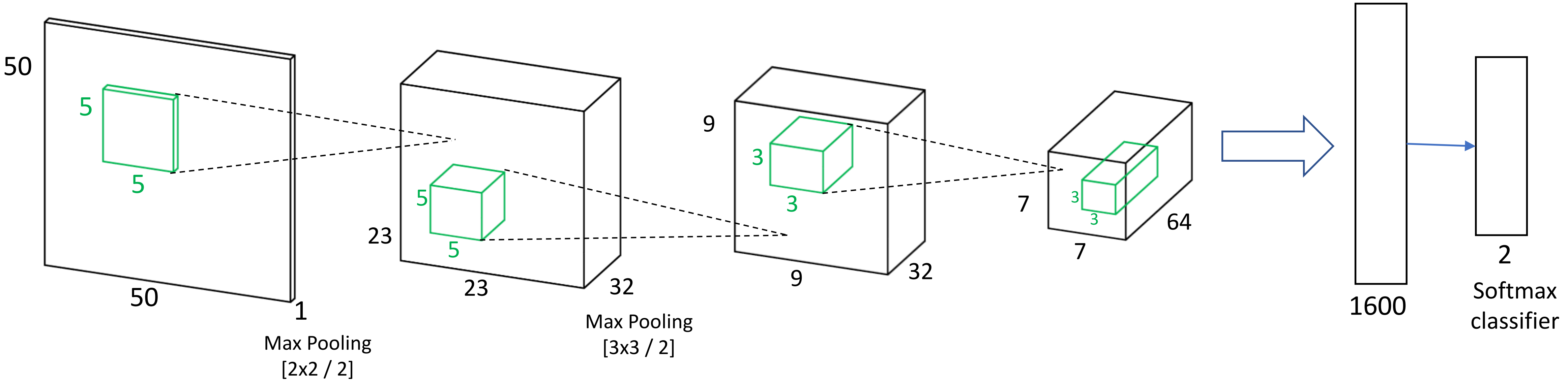
Malignant



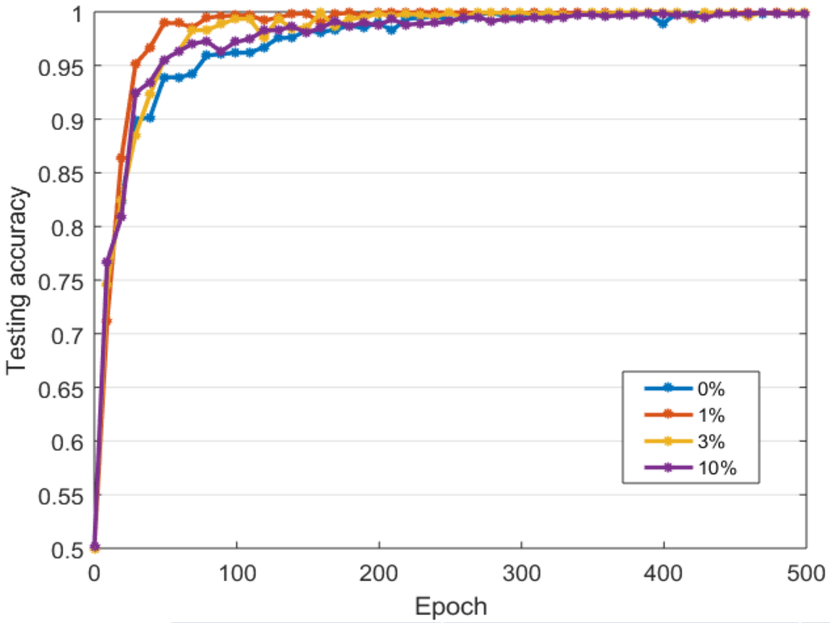
- Resulting displacement field was obtained by *compressing* the tissue virtually in Finite Element Solver at low strain for each SM distribution to generate the data-set of 10,000 distinct displacement fields, which acts as an input to the CNN.
- In reality, the measured displacement field is often corrupted with the noise. To simulate this different level of Gaussian noise was added to the resulting displacement field.

Problem set-up

CNN Architecture :



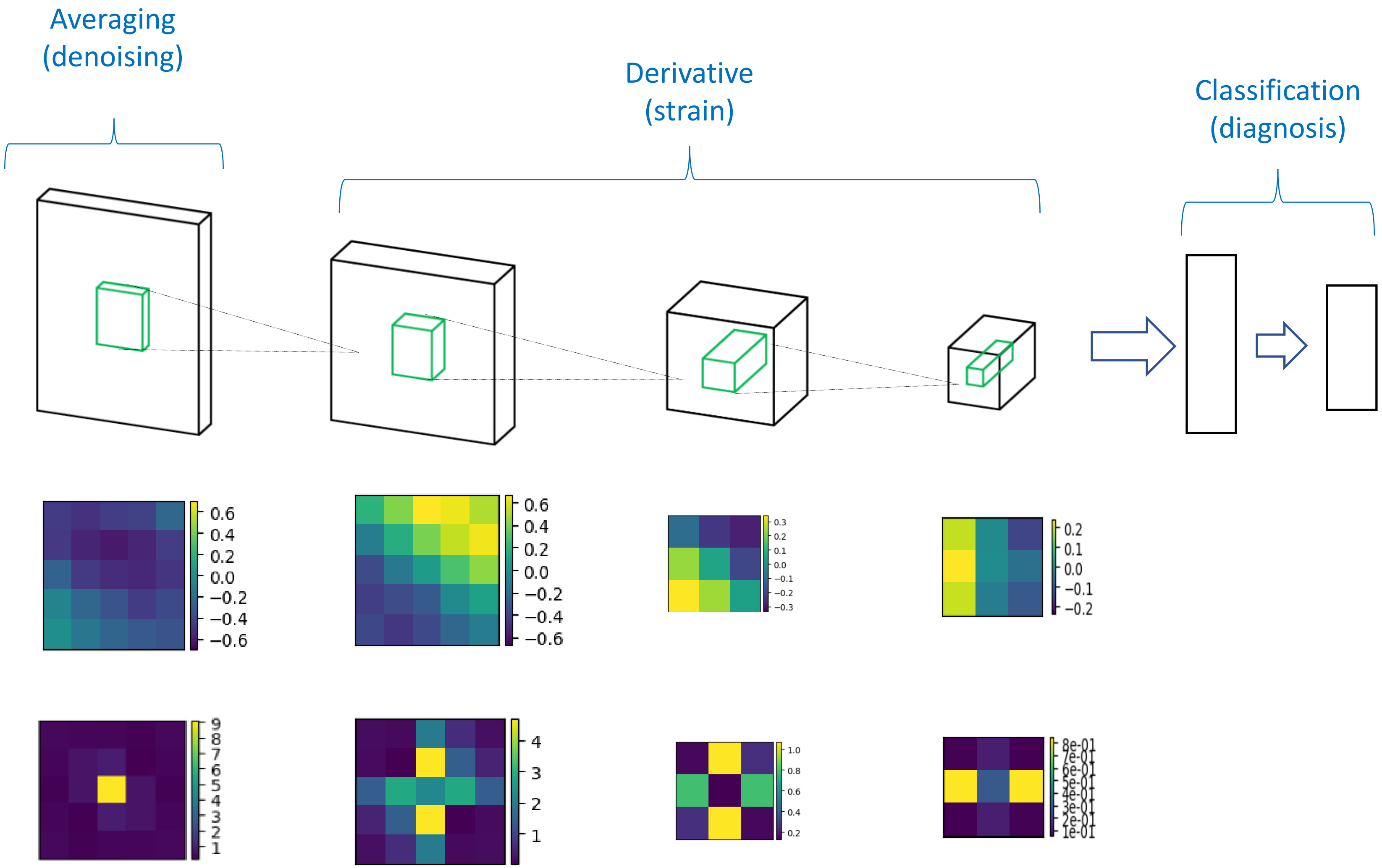
Results [heterogeneity study]



Confusion matrix (N=2000) [1% noise]		Actual	
		Benign	Malignant
Predicted	Benign	1000	2
	Malignant	0	998

Noise level (in %)	Accuracy	Specificity (TN/N)	Sensitivity (TP/P)
0	99.95%	100%	99.9%
1	**99.9%**	**100%**	**99.8%**
3	99.95%	100%	99.9%
10	99.75%	99.7%	99.8%

Connection to traditional Elastography



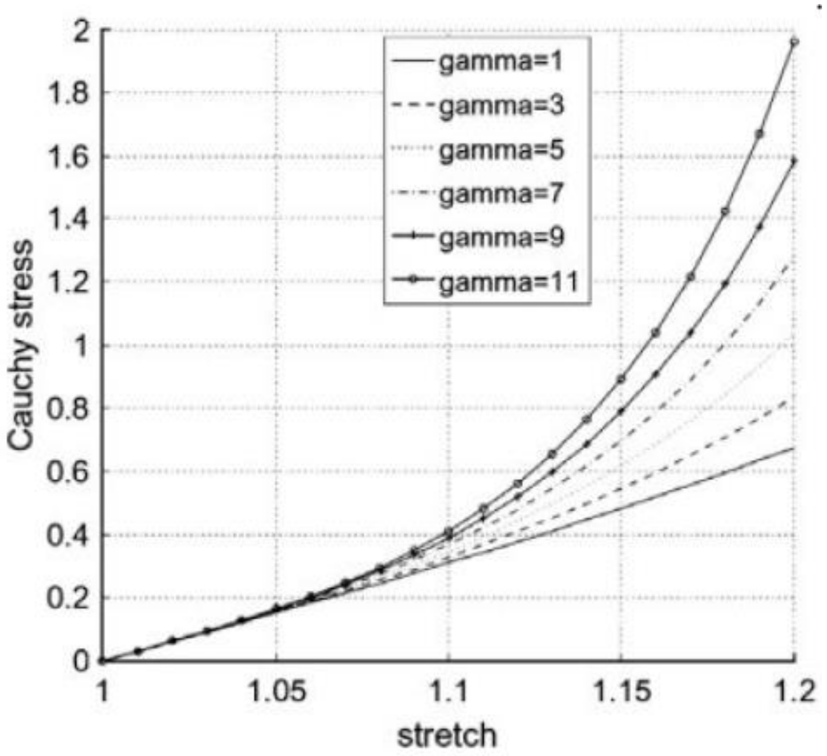
Convolution filters
(Discrete differential operators)

Fourier Transform of Conv. filter

Problem Setup

We model the tissue as an incompressible isotropic hyper-elastic solid with strain energy density function given by:

$$W = \frac{\mu}{2\gamma}\left[e^{\gamma\left(J^{\frac{-2}{3}} I_1 - 3\right)} - 1\right]$$



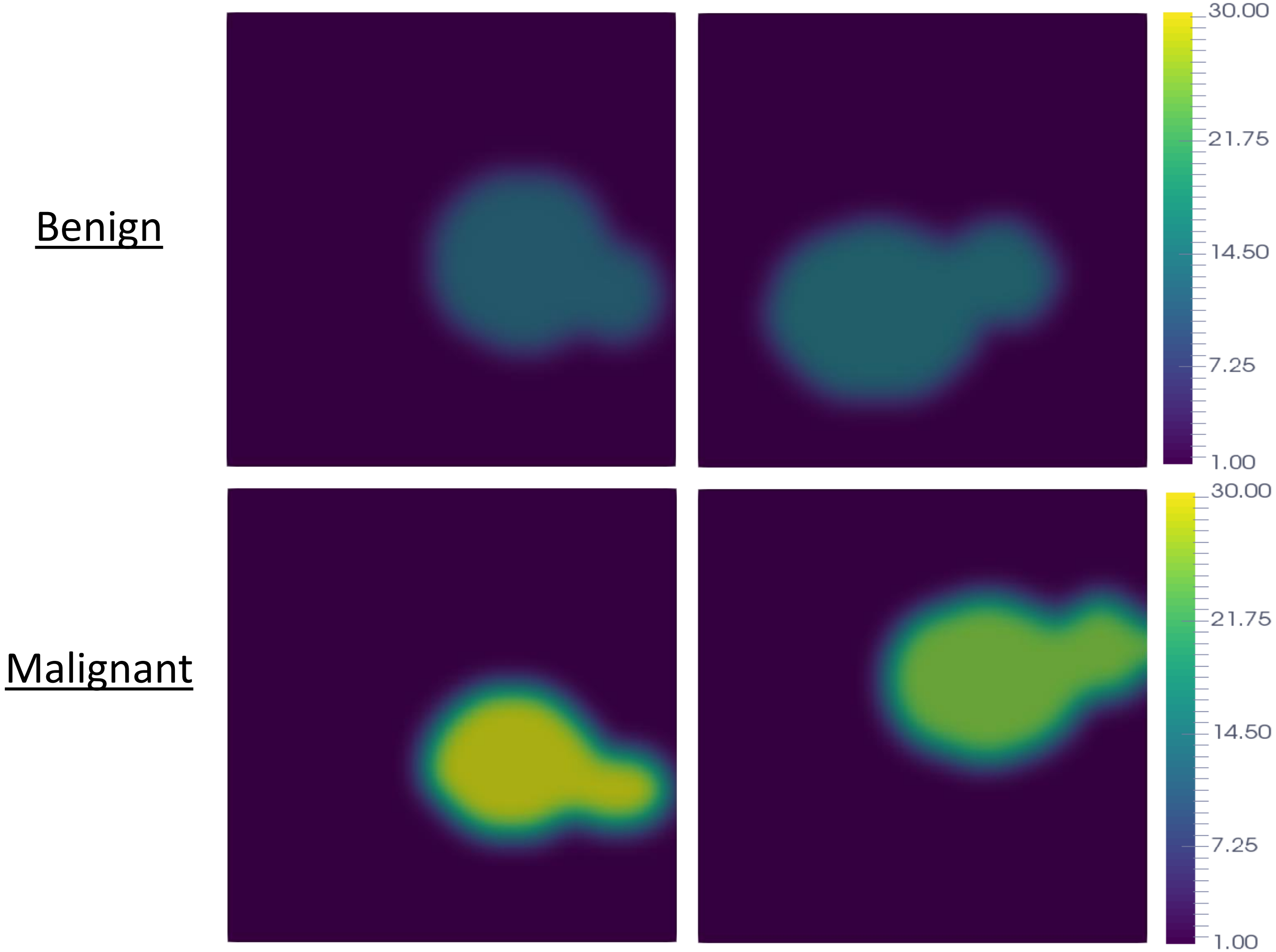
- μ represents the slope of the stress-strain curve at zero strain.
- γ represents the nonlinear elastic response of the material

Nonlinearity study

It is observed that the malignant tumors tend to stiffen at faster rate than their benign counterparts – have higher average value of nonlinear parameter(γ).

Problem setup

Non-linearity study



- For this study, the shape of the tumor for both the cases is equal but the value of non-linear parameter is changed.

- For benign case, the value of NLP is in the range of 5-15, whereas for the malignant case it is in the range of 20-40. This makes the latter elastically more non-linear.

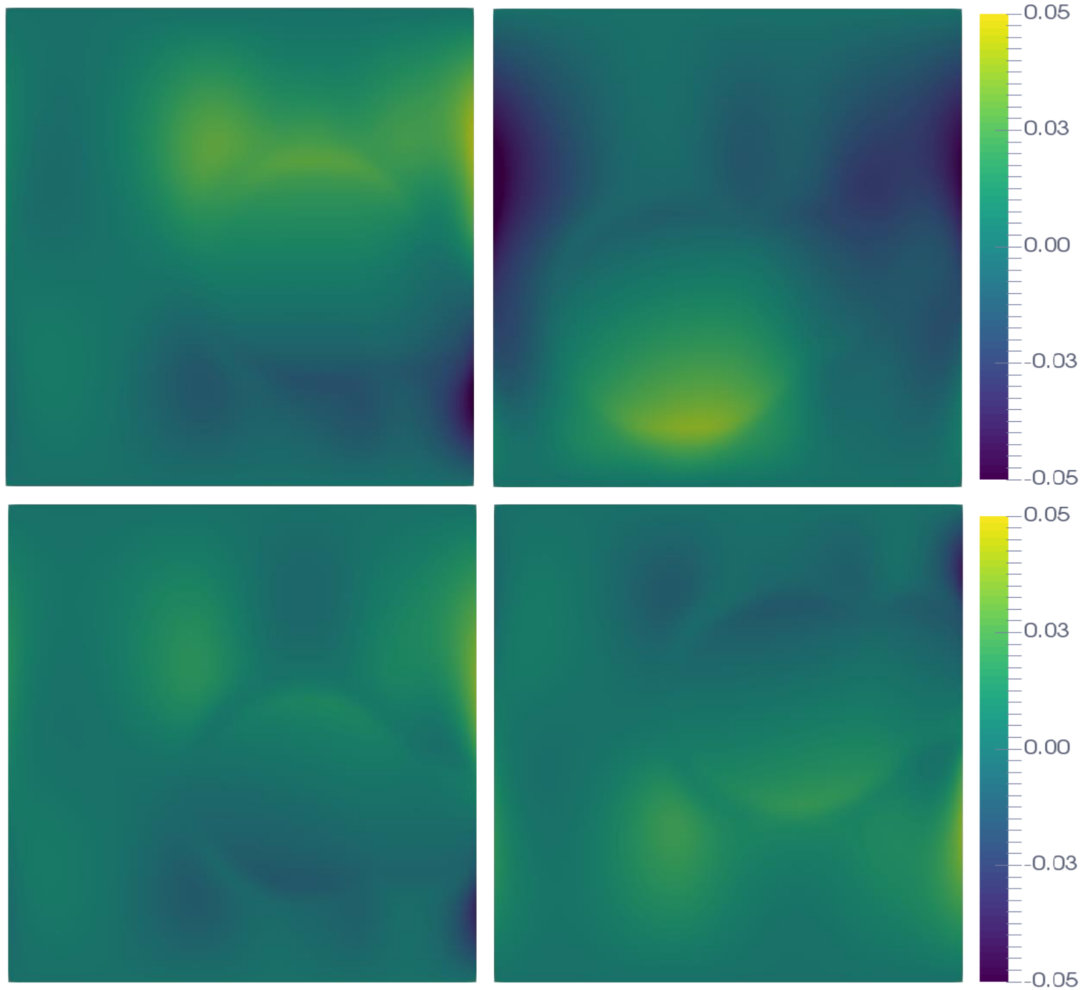
- 5000 different NLP distributions were generated (2500 of each class) by changing the value of NLP.

Problem setup

Generating *difference of displacement images* [Nonlinearity study]

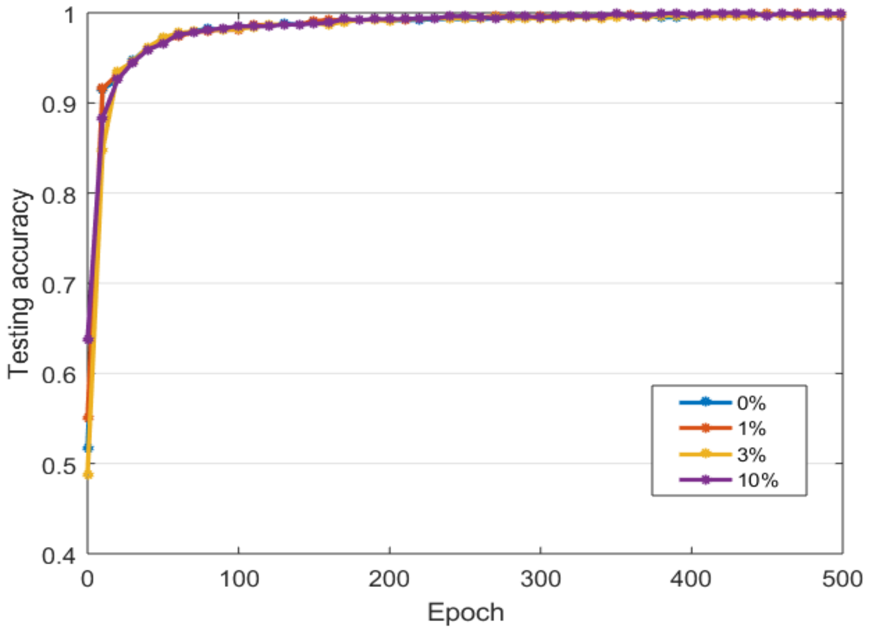
Benign

Malignant



- For each tumor the tissue is first compressed at 1% strain and then at 20% strain to obtain displacement field at two different strain level.
- These two displacement fields are then normalized and subtracted to obtain “difference of normalized displacement images”, which acts as an input to our CNN.
- Again, the effect of noise in the displacement field on the performance of net was tested by adding different level of Gaussian noise.

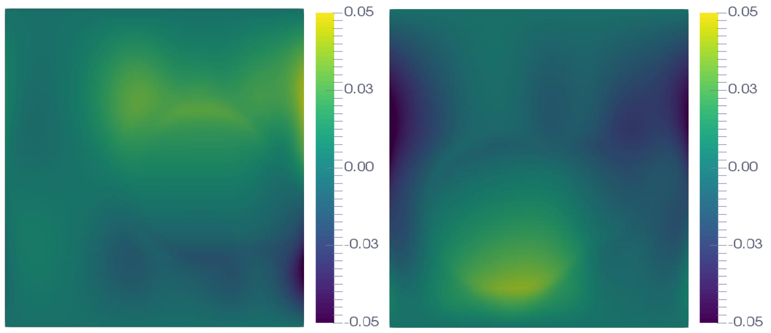
Results [nonlinearity study]



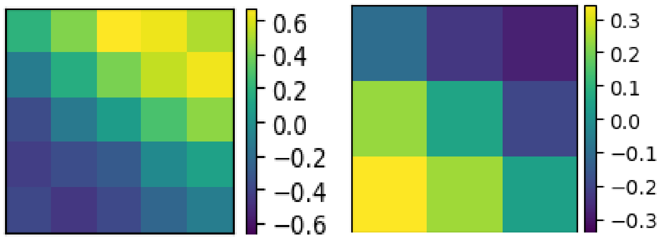
Confusion matrix (N=1000) [1% noise]		Actual	
		Benign	Malignant
Predicted	Benign	498	1
	Malignant	2	499

Noise level (in %)	Accuracy	Specificity (TN/N)	Sensitivity (TP/P)
0	99.8%	100%	99.6%
1	**99.7%**	**99.6%**	**99.8%**
3	99.8%	99.8%	99.8%
10	99.9%	99.8%	100%

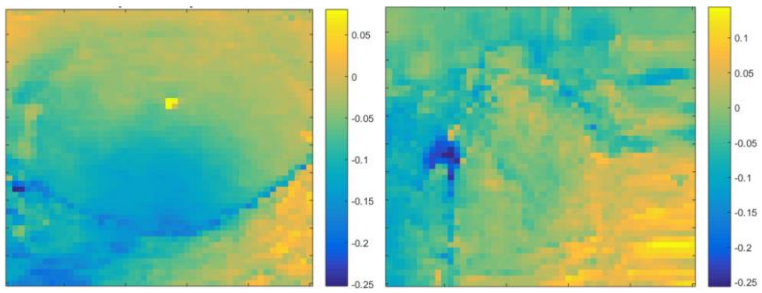
Testing on real patient data



Train the CNN entirely using ***synthetic data***



Learn the parameters of the model



Doing prediction on a ***real data*** using learned weights

Diagnosis : Benign/Malignant

Testing on real patient data

- Preprocessed the patient data to make it consistent with the input of the CNN.
- Used the learned weights from 3% noise model of nonlinearity study to do the prediction on 10 patients.
- Physics based transfer learning approach, where we introduced the physics through synthetic training data seems to be giving promising results.

Confusion matrix (N=10)		**Actual**	
		Benign	Malignant
Predicted	Benign	4	1
	Malignant	1	4

Conclusion

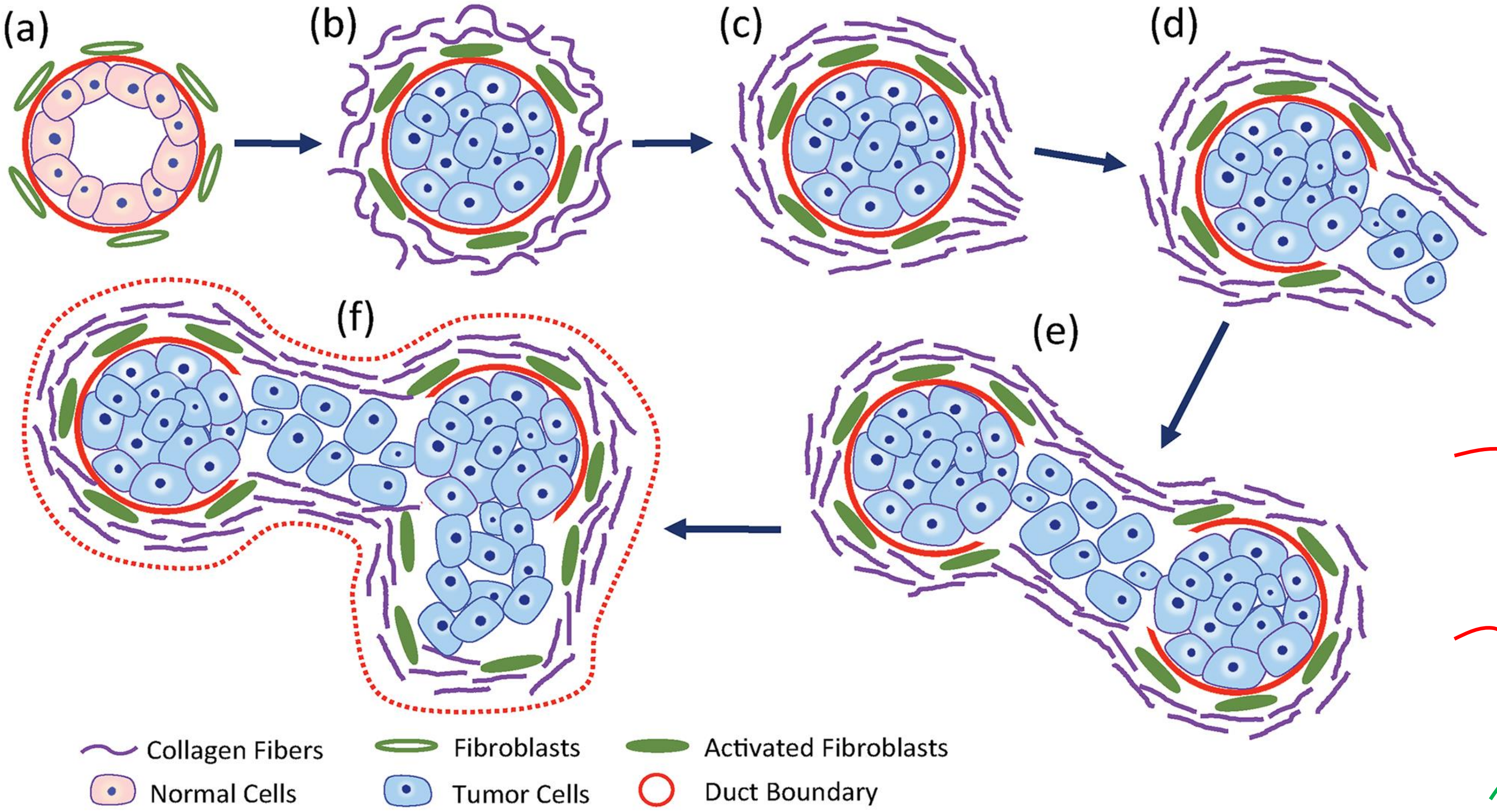
- Bypassed the solution of an expensive and ill-posed inverse problem.
- Demonstrated the robustness of this data driven model to highly noisy measurements.
- Explored the connection between this learning based approach and traditional Elastography by analyzing the convolution.
- Physics based transfer learning.
- Possibility of getting even better accuracy by using hybrid dataset.


THANK
YOU!

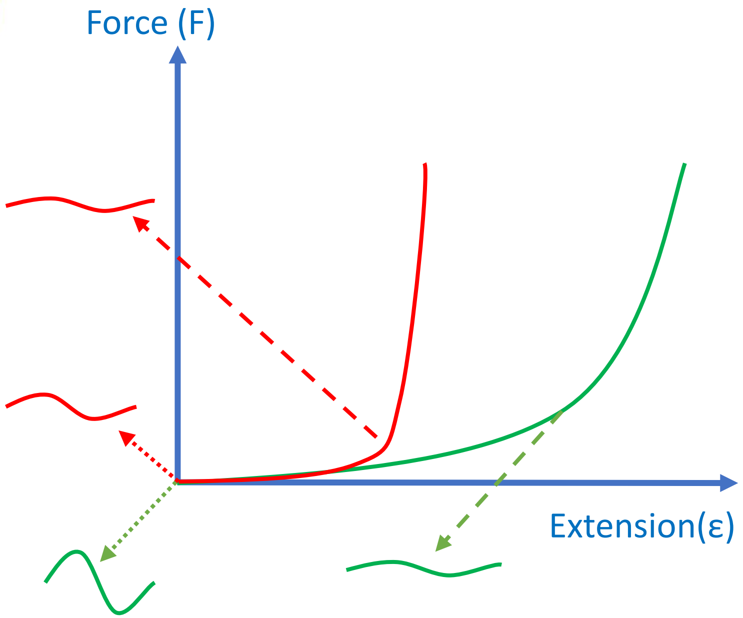
Extra Slides

Mechanics of cancer

Cartoon of tumorigenesis [Liu et al. 2015]



- Increased collagen
- Less tortuous
- Heterogeneous



Slide credit : Dr. Oberai

Inception(v3) model

Heterogeneity Study

Noise Level (in %)	[Total images = 600 (300b,300m)]		
	Train	**Val**	**Test**
0	100%	98.4%[61]	96.4%[56]
1	99%	95.2%[63]	98.6%[72]
3	96%	91.8%[61]	95%[60]
10	88%	57.1%[63]	69.4%[62]

Nonlinearity Study

Noise Level (in %)	[Total images = 300 (150b,150m)]		
	Train	**Val**	**Test**
0	100%	100%[31]	100%[31]
1	100%	100%[30]	95.7%[23]
3	100%	100%[31]	100%[29]
10	100%	100%[27]	100%[25]

No. of validation and testing images are shown in the bracket.